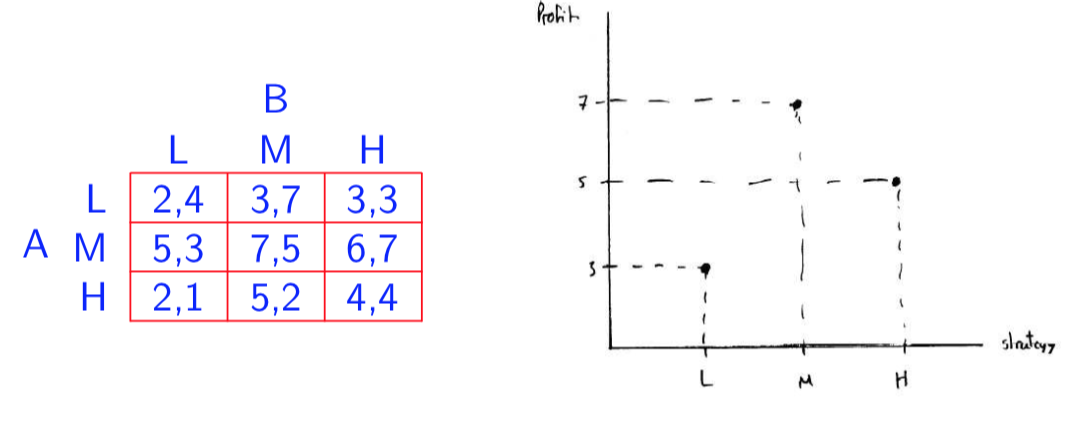
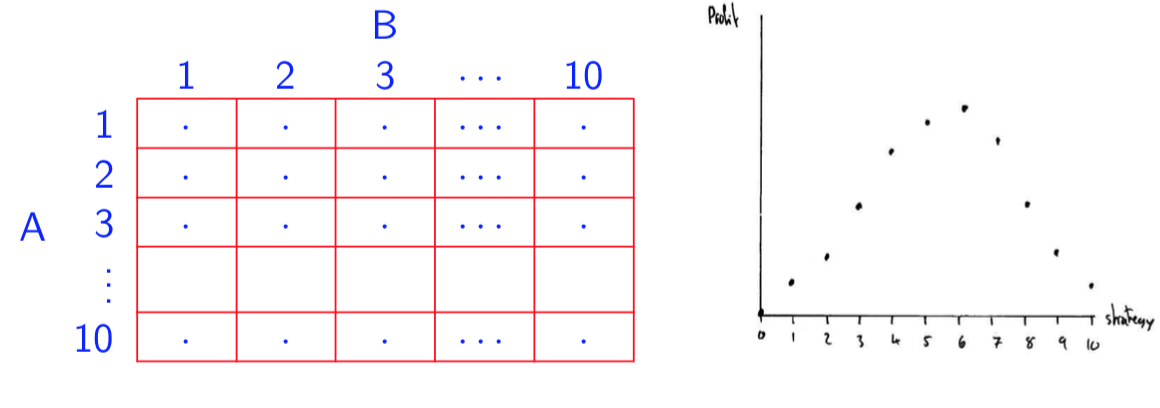
**4: Games With Continuous Strategies**

* This is applying maths to what we already know
* Nash Equilibriums & Sub-Game Perfect Nash Equilibriums remain the same
* This is applying the following more generally
* Matrix strategy: can choose any option for the expected opponent’s options
  + Recall:
  + Simultaneous: Best Responses & Mutually Consistent Best Responses
  + Sequential: Backward Induction
* Take a long time to analyse a Continuous Strategy using Discrete Sets (matrix)

**4.1: Quantity Competition**

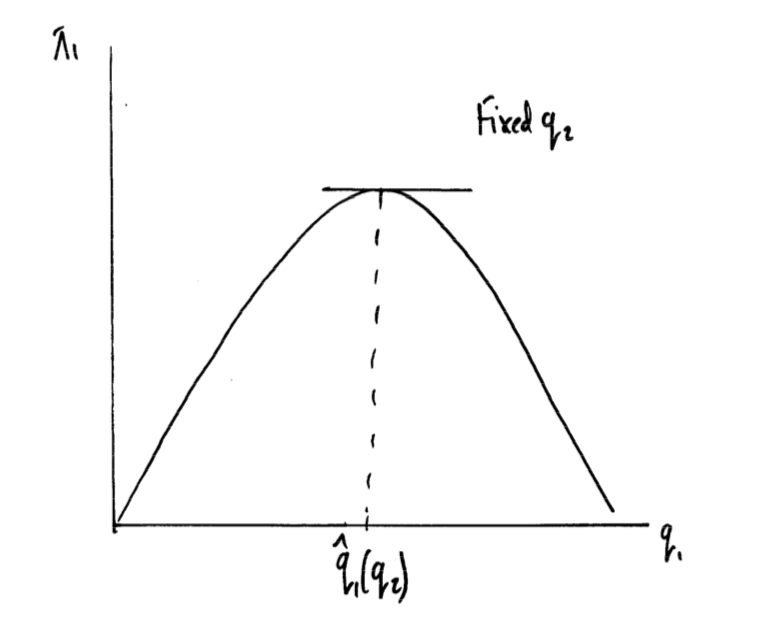
* In a **Competitive Market**
* Firm *i* supplies
  + Where Total (Aggregate) Quantity: *Q*
* Inverse Demand Function:
* Payoff is Profit ():
* Hence:
* Oligopoly if several firms compete
* Recall that A could choose any option from 1 to NMatrix in response to B
  + “ could be anything from 1 to NMatrix”
* This can be reflected in chart form but can prove difficult in high Ns:



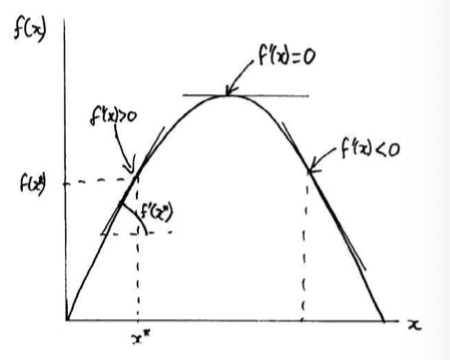
**…**

* + Hence, we see a Payoff Function which is maximised at a point
  + “Find the level of maximiseing firm 1’s payoff for given ”

**4.2: Continuous Strategies**

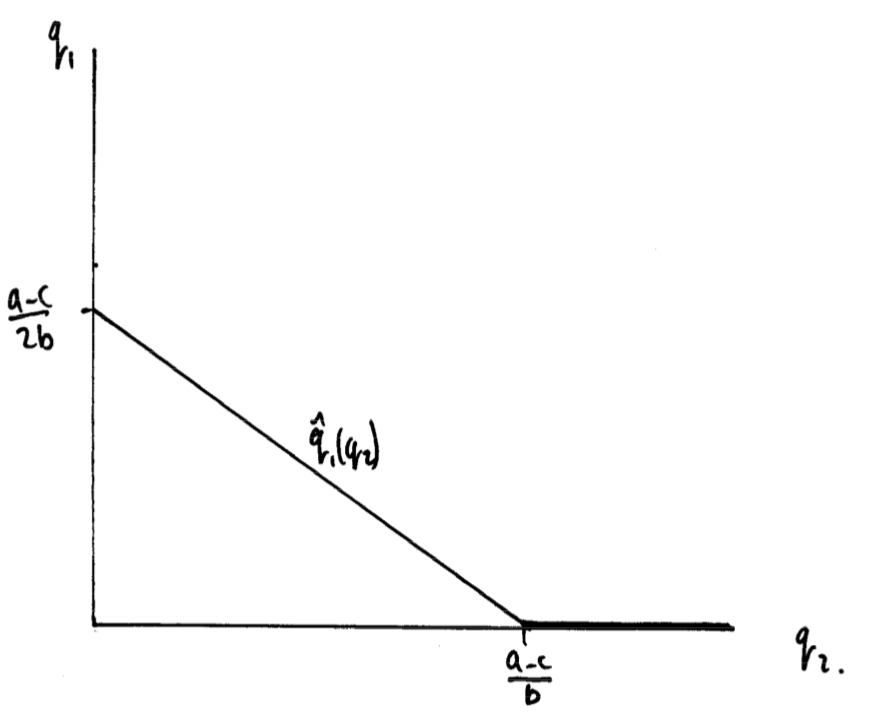
* Too hard to account for all the options (in this case quantities to produce)
* Recall Basic Maths:
  + Function: the level
  + Derivative: the slope of the function
  + Partial Derivative: fix a variable (extract from equation {Hyp. = 0})
* Recall **Rules of Differentiation**:
  + Working towards Payoff Function
  + Just like in the matrixes, fix the opponents option each time to find your best
  + Hence,
  + Fix (the other’s strategy) to observe how varies with
  + Therefore, **partial derivative:**  for fixed
  + Thus, Best Response at {} (peak of function)
  + Note that, if you take the derivative on the **incline** of the function, you can be made better off by doing more. Take the derivative on the **decline**, better off by doing less
  + 

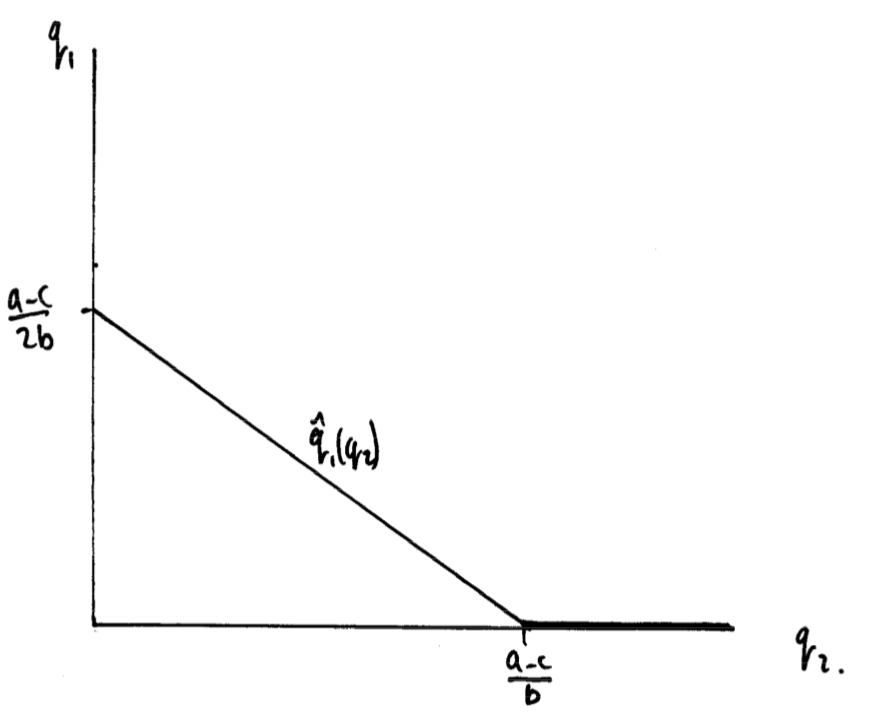


* + Recall: the function :
  + Recall: if
    - **Constant**:
    - **Sum**:
    - **Product:**
    - **Chain:**
    - **Quotient:**
    - **Log:**
  + In Practice:
    - **Power**:
    - **Constant**:
    - **Sum**:
    - **Product:**
    - **Chain:**
    - **Quotient:**

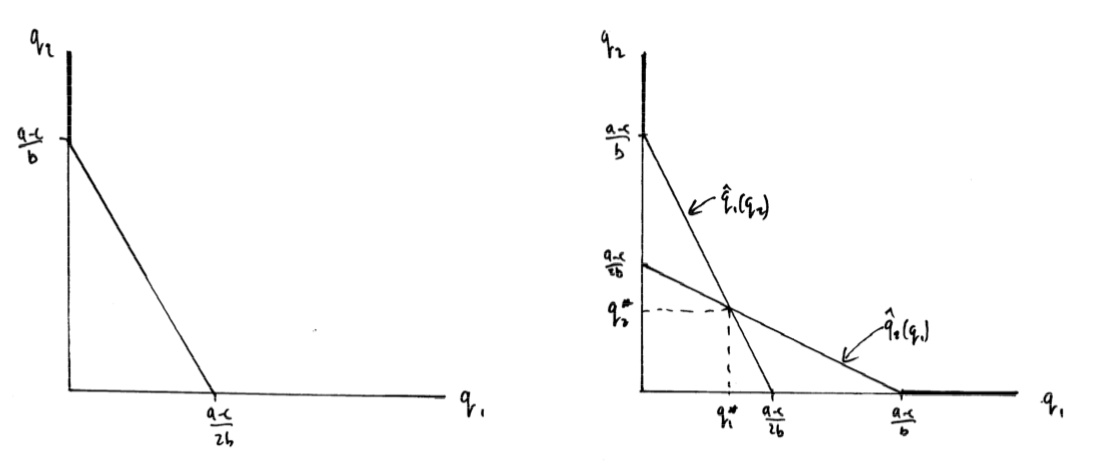
**4.3: Cournot Derivation of Payoff & Reaction (Simultaneous)**

1. Fix **Firm 2**’s action and find my Best Response through Payoff Function
   * Find Payoff Function
   * Partially Derive & {=0} for Best Response with fixed
   * Find best for Reaction Function; Sub for
2. Fix **Firm 1**’s action and find their Best Response through Payoff Function
   * Find Payoff Function
   * Partially Derive & {=0} for Best Response with fixed
   * Find best for Reaction Function; Sub for
3. Find meeting point of **Nash Equilibrium** where both firm’s Reaction Functions meet
4. (Optional) Substitute to find the optimal π for each firm

* **Players**: 2 firms of
* **Strategies**: each firm chooses quantity of
  + For Quantity
* **Payoff**: given supply choices,
  + Marginal Cost of
  + Price
  + Payoff (Profit)
* Working Example for **Firm 1**:
  + For :
  + (Payoff Function of Firm 1)
  + (Fixed )
  + … (Reaction Function of Firm 1)
  + Recall:
  + Note that Reaction Function:
  + Hence, Reaction Function:
  + Output quantity should decline as the opponent’s increases
  + When it reaches 0, leave market
  + Obviously no negative
* Repeat for **Firm 2**:
  + (Payoff Function of Firm 2)
  + (Fixed )
  + or (Reaction Function of Firm 2)
  + Note that Reaction Function:

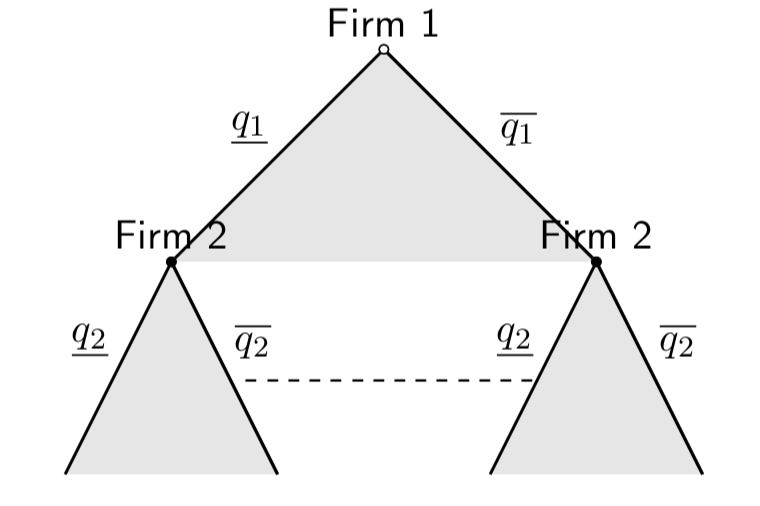


* **Nash Equilibrium**:
  + “The Cournot Equilibrium”
  + Flip Firm 2’s Reaction Function and overlay
  + Seek:
  + “For Firm 1’s *q* which maximises its π given Firm 2’s *q*”
  + “For Firm 2’s *q* which maximises its π given Firm 1’s *q*”



* + Achieved through **Substitution**
  + From:
  + Sub for Firm 2…Sub for π’s…
* Cournot Equilibrium \*\*
  + Hence:
  + So:
  + Thus:
  + ;; ;
* Verify that:
  + Industry Output Between Monopoly and PC:
  + Price is Between Monopoly and PC:
  + Industry Profit Between Monopoly and PC:

**4.4: Stackelberg Leader & Follower (Sequential)**

* The leader implements the first player’s Reaction Function intro their Payoff Function
* First mover advantage as leader gets higher payoff
* Recall from Sequential Games: Backward Induction
* **Linear Demand**:
* **Constant Marginal Costs**:
* **Profits**:
* Firm 1 moves, Firm 2 observes and moves
* Firm 1: Leader; Firm 2: Follower
* Recall Observation:
* Backward Induction:
  + Stage 2: Firm 2 maximises profits given

Firm 2 uses Best Response to whatever Firm 1 produces

* + Stage 1: Firm 1 anticipates reaction of Firm 2 to any decision made

Firm 1 maximises profits given response of Firm 2

Firm 1 chooses point on Firm 2’s Reaction Function which maximises profits

* **Stage 2:**
  + Given what’s the best for Firm 2 (follower) to do? – As Previously…
  + Recall: (Payoff Function)
  + Optimise and {=0}:
  + React: (Reaction Function)
* **Stage 1:** 
  + Firm 1 (Leader) will choose to max. profits taking into account the reaction of the follower
  + Recall: (Payoff)
  + It Knows: if they choose Firm 2 will respond with – account
  + Firm 1 (Leader):

* So Firm 1 Maximises:
* Therefore: (Reaction)
* So Firm 2 (Follower): (Reaction)
* Stackelberg Equilibrium \*\*
  + Idea Is: rather than equilibrium, there is an advantage
  + ; ;
* Stackelberg vs. Cournot:
  + Cournot: ; ; ;
  + Stackelberg: ;

;

; ;

* Hence, First Mover (Firm 1) Advantage!

**5: Applications of Prisoner’s Dilemma**

**5.1: Recalling The Cournot Game**

* An example of a continuous game
* Rather than using Reaction Functions, find Isoprofit Curves
* This is like indifference curves for firms
* Call Firms:
* Strategies:
* Payoff:

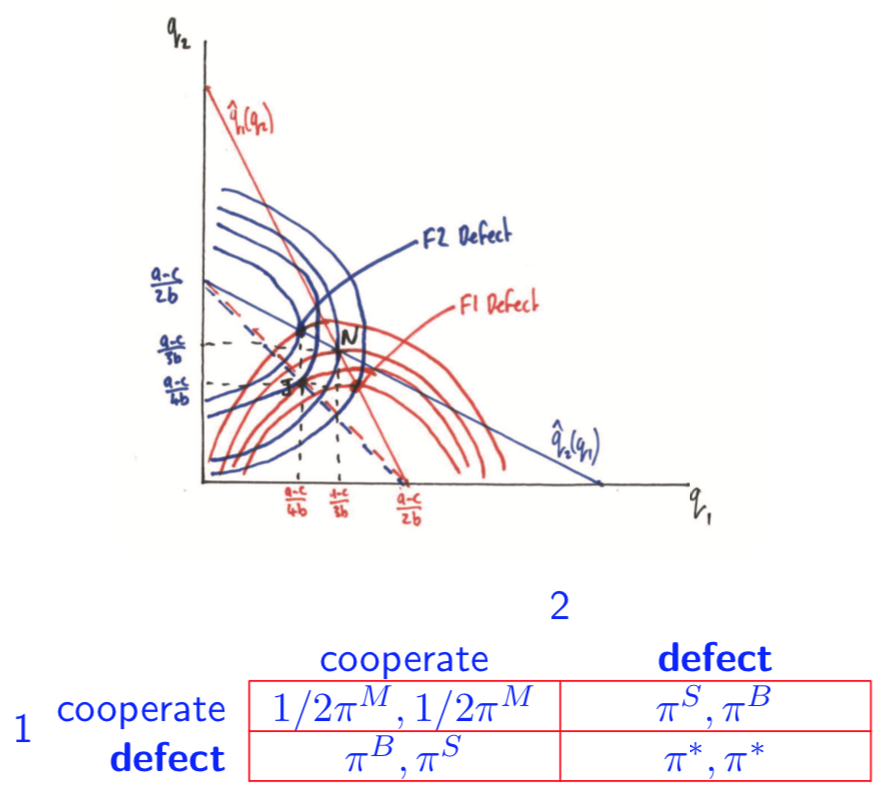
***5.1.1: Typical Reaction Function***

* + From Isoprofit contours
  + Equilibrium at intersection:

***5.1.2: Maximising Joint Profit***

* Hence: assuming ;
  + Makes sense as:

***5.1.3: Will Firms Agree?***

* Will firms agree to produce at half the monopoly output?
* **No**: if firms expect you to produce more than ;
  + Best possible: 🡪 must expand output in excess of Cournot Output
  + Defecting firm: **Bonanza** Payoff
  + Cooperating firm: **Sucker** Payoff
* Hence: Prisoner’s Dilemma

**5.2: Externalities & Strategic Nature**

***5.2.1: Externalities***

* **Negative**:
  + You do more, you lower my payoff (Cournot Game)
  + (Slope of Payoff Function)
* **Positive**:
  + You do more, you lower my payoff
  + (Slope of Payoff Function)

***5.2.2: Strategic Nature***

* **Strategic Substitutes**:
  + Opponent does more of their action: you optimally do less (Reaction Function downward)
  + “with a higher *xj* the optimum is with a lower *xi*”
* **Strategic Compliments**:
  + Opponent does more of their action: you optimally do more (Reaction Function upward)
  + “with a higher *xj* the optimum is with a higher *xi*”

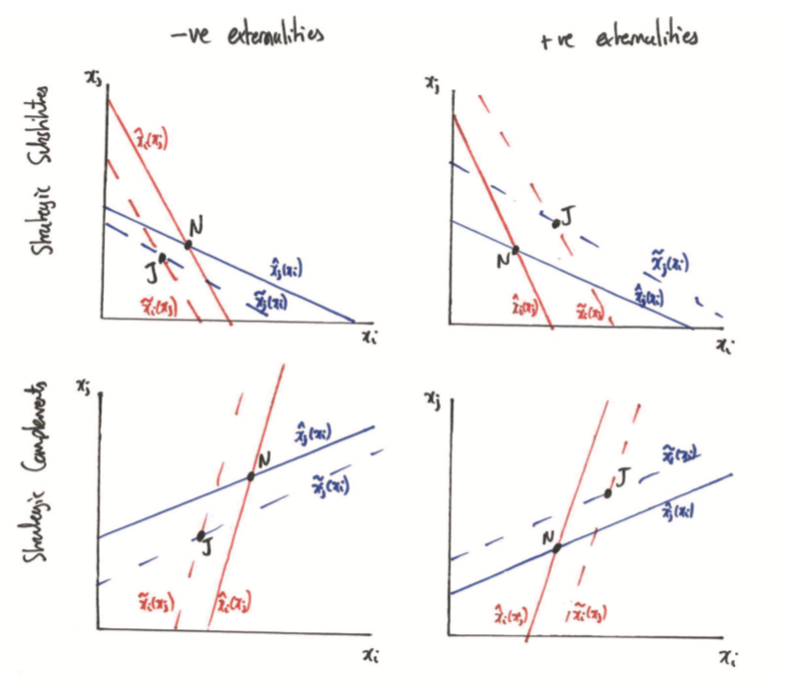
**5.3: Nash Equilibrium in Games**

* When non-cooperative, players optimise self-interest
* Marginal Payoff = 0: &
  + *Note that hat implies function*
* Nash Equilibrium at: Equilibrium
  + Thus Nash Equilibrium actions:

**5.4: Social Planner**

* What happens when they ‘internalise’ the externality?
* Social Planner maximises joint payoff
  + - Chooses to maximise
    - 🡪 these are both Social Optimums

**5.5: Nash Equilibrium vs. Optimum**

* w/ **Positive Externalities**
  + {}
  + So for Social Planner {}
  + So:
* w/ **Negative Externalities**
  + {}
  + So for Social Planner {}
  + So: