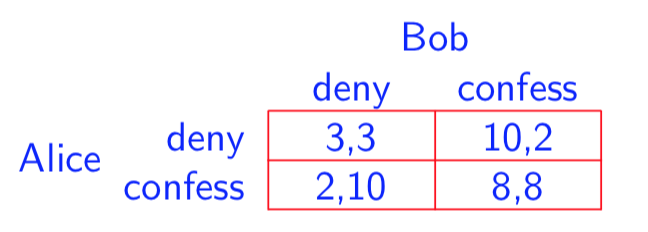
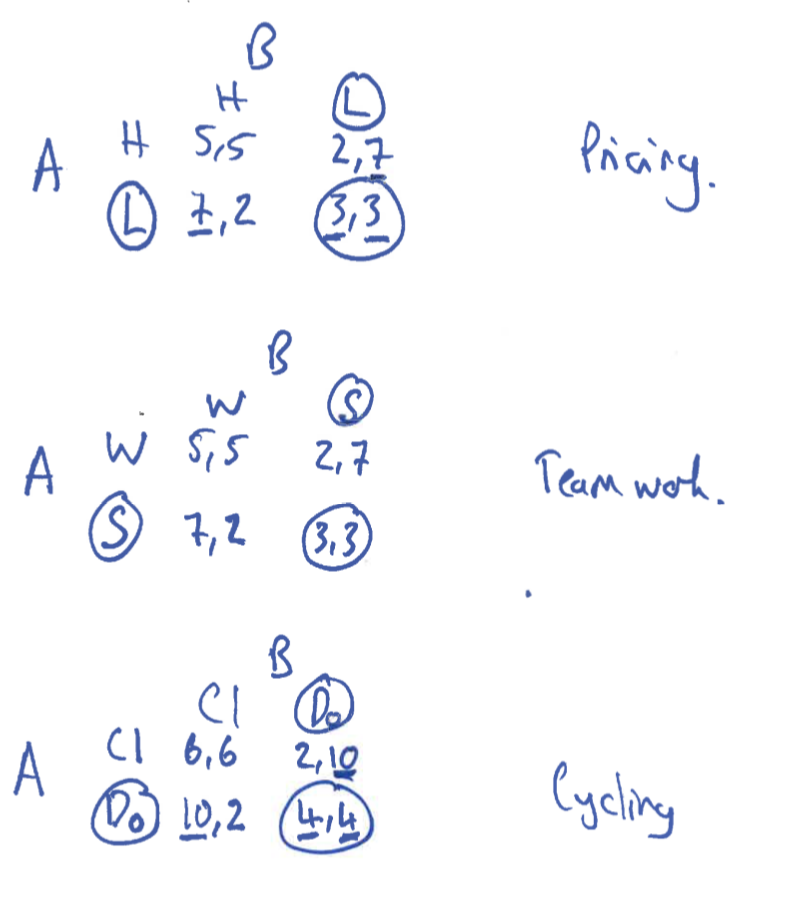
**3: Prisoner’s Dilemma**

* Cooperate or Defect
* Mutual Gain: Cooperate
* Individual Incentive: Defect
* Pareto Inefficient Equilibrium
* Recall: someone has a Dominant Strategy where there’s harm done to each other and they could be better off (Pareto Efficient). Self-interest doesn’t pay off. But:
  + Games can be repeated (e.g. price re-setting)
  + E.g. lower price than opponent now (get more custom volume), makes opponent less-off (also poor for aggregate prices)
  + Firms may form a passive collusion where they both think opponents will set low so they both set high
* Both players have Dominant Strategy to Defect but they could have a better result when the both Cooperate
  + ****Hence, when choosing best interest, harm is done to the opponent when choosing to Defect for own interest, the opponent may choose to Cooperate so have a worse outcome

**Example (1):** (8,8) is Pareto Inefficient equilibrium as it is reached by both aiming for low by confessing. Could be made better off by both acting for mutual gain (3,3)

**Example (2):** **Pricing** – non-brand loyal market, flow freely between

**Example (3): Team** **Work** – (work vs shirk) shirk leads to more payoff as still full marks but no work done but if the other does all the work, they will get full marks but payoff will reduce due to workload

**Example (4): Clean** **vs** **Dope** – (risk based) best self-interest response is to dope as highest possible payoff but the equilibrium they both do it is less than the payoff if they both don’t

**Example (5): Market Share** – studying marketing is a waste of time. Market is a pie, we compete over our share. Ads try to (1) inform & (2) predatory (winning market share). Start: 50/50, engage in ads to win market share. I spend money, I get some in return but you won’t gain much more market share. The opponents do this to keep up. Each keep catching back up to 50% each but both are still wasting millions on marketing. Market share isn’t changing proportionately but you’re still spending money

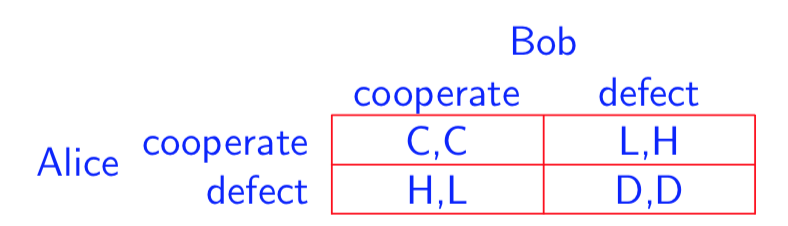
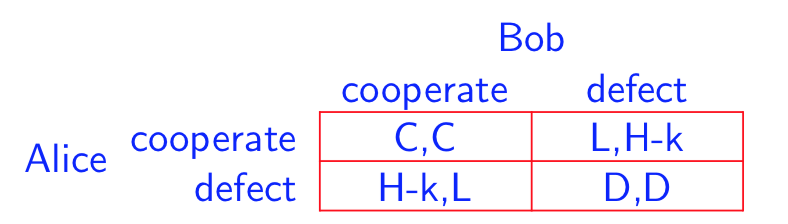
**3.1: Externalities**

* Negative Externalities: (own interest – doing too much) Cooperation reduces amount of work you do for the better (e.g. not over-fishing)
  + Don’t see costs from defecting – too much harmful activity is done
* Positive Externalities**:** (own interest – doing too little) Cooperation says you should do more work (e.g. not doing no work in a project)
  + Don’t see costs benefits form cooperating – too little of a good activity is done
* **Example**: Marginal Benefit vs. Marginal Cost – (1) extracting fish from the ocean makes it harder in the future (e.g. do less fishing to allow repopulation). (2) But you want more to sell now. (3) Self-interest makes it harder for others

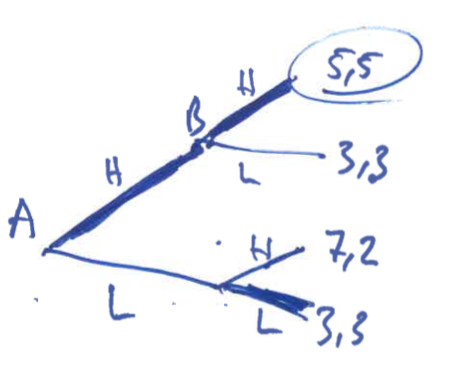
**3.2: Rationalise Cooperation Resolutions**

* Resolutions to Prisoner’s Dilemma
* **Meet-up** (verbal agreement): let’s set high prices (incentive of deception however – you want him to set high prices and you want to set low)
* **Threaten**: punishment of opponent doesn’t set high prices (lacking credibility as you “will do it” rather than it “will be done (automatically etc.)” – can fix credibility by using Mafia as they have more incentive to harm him)
* **Reward**: offer a reward that overcomes the incentive to Defect (still lacks credibility as it involves giving money – lowers your payoff. May not even believe you)

**3.3: Behavioural Resolution**

* People are influenced by the ‘social norm’: if people conflict with this, there’s a cost
  + Denote this cost of Defecting as ***k***
  + Recall:
  + H > C > D > L
  + So:
  + If ***k*** large enough relative to ***H – C***, behaviour in defecting contrasts with ‘social norm’ so cost
* External Norms of Behaviour:
  + Think back to litter e.g.: many people won’t actually litter even though it’s the most beneficial for you. It conflicts with the social norm
* Internal Norms of Behaviour:
  + Doing nice things for people who are nice to you (gain utility)
  + Being bad to people but they are good to you (loss of utility). - you defect but if you care enough, you’ll maybe rationalise Cooperation and change mind

**3.4: Price Matching Guarantee Resolution**

****

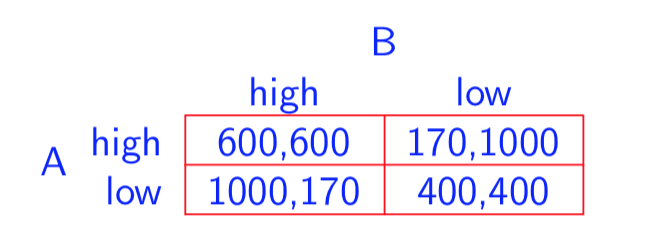
* You may be undercut for opponent to gain market share from you at lower prices
* If you’re offered. PMG, you will simply match prices and keep customers “he’s selling at that price, can you just sell me at that too”
* Opponent now doesn’t gain, just sells at lower price as no market gain
* Equilibrium of both pricing high – out of Prisoner’s Dilemma

**3.5: Dynamic Punishment Resolution**

* When Defecting, a player may believe they will be ‘punished’ in the future
* Can we achieve Cooperation through fear of Punishment?
  + **Credible:** backed with fact
  + **Incredible:** maybe won’t happen
* **Finite Period** (*T* Periods): Defect in last period as no more time for **retaliation**
  + Final period: mutual Dominant Strategy to Defect as no future punishment
  + So: best to defect this period as well as you both will next
* **Infinite Period:** the game will continue [probability p=1] so **retaliation**
  + Always an opportunity to punish as there’s always another period
* **Impatient:** future worth less than present so Defect (not caring for punishment)
* **Patient:** care more for future gain by waiting and Cooperating

***3.5.1: Discounting***

* **Stream of Payoffs:**
* £1 from £1 today to £1(1+*r*) tomorrow; £1 from £1 tomorrow to today
  + **PV**: ; ***r*** = Discount Rate; = Discount Factor =
  + **Hence**: £*X* in period *t* is worth today
  + **Hence**: £*X* in period *t* is worth today
  + Close to 1: **Patient;**  Close to 0: **Impatient**
  + ;
  + **Infinite:**
  + **E.g.** Payoff of 7 in perpetuity: Payoff of 10 today and 2 in perpetuity:
  + **;**

**Note:**

***3.5.2: Trigger Strategies – Grim Trigger***

* Start by Cooperating
* If opponent Cooperated, Cooperate
* If opponent Defected, Defect in perpetuity

1. Cooperate:
   * Opponent gets 600 forever 🡪
2. Defect:
   * Get 1000 now but 400 after 🡪
3. Answer:
   * Hence: **Grim Trigger** at so Cooperate

***3.5.3: Trigger Strategies – Tit-For-Tat***

* Start by Cooperating
* Play as the opponent played in the last round
* Cooperation followed by Cooperation
* Defection followed by Defection

1. Defect in Perpetuity:
   * Same as **Grim**
2. Defect Once:
   * Get 400 now but loses 430 after 🡪
3. Answer:
   * Hence: **TFT** at so Cooperate

If **Grim** Works: Cooperation is **possible**

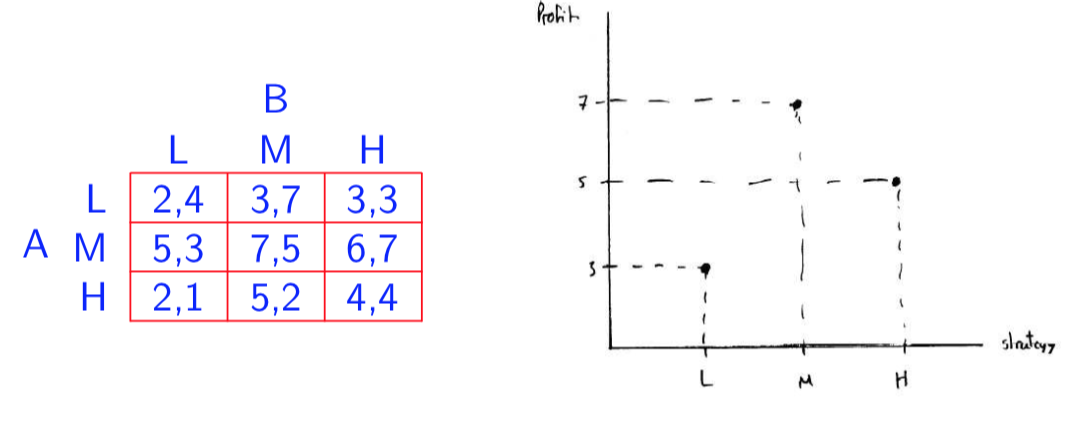
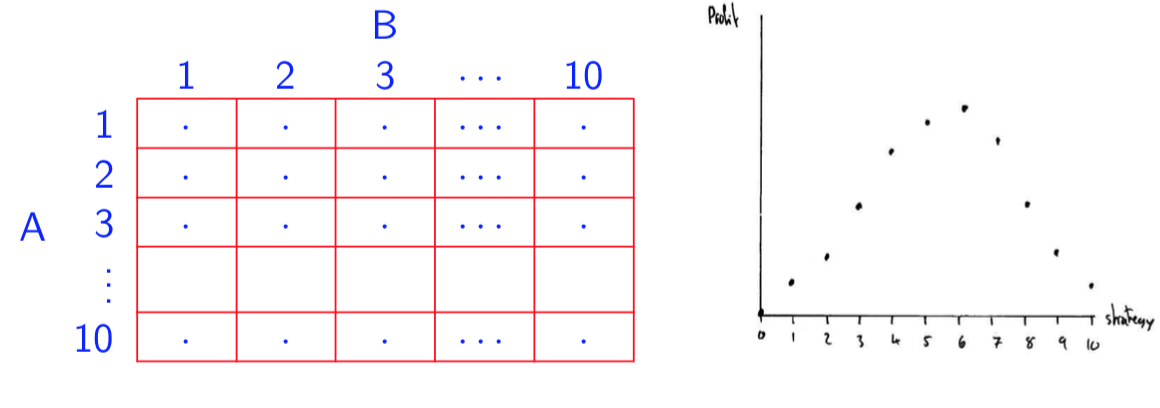
If **TFT** Works: Cooperation is **easy**

**4: Games With Continuous Strategies**

* This is applying maths to what we already know
* Nash Equilibriums & Sub-Game Perfect Nash Equilibriums remain the same
* This is applying the following more generally
* Matrix strategy: can choose any option for the expected opponent’s options
  + Recall:
  + Simultaneous: Best Responses & Mutually Consistent Best Responses
  + Sequential: Backward Induction
* Take a long time to analyse a Continuous Strategy using Discrete Sets (matrix)

**4.1: Quantity Competition**

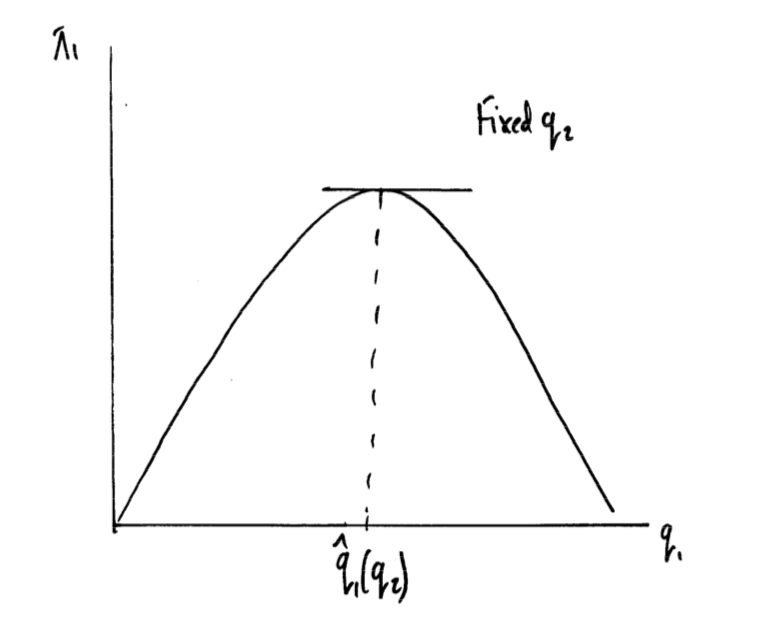
* In a **Competitive Market**
* Firm *i* supplies
  + Where Total (Aggregate) Quantity: *Q*
* Inverse Demand Function:
* Payoff is Profit ():
* Hence:
* Oligopoly if several firms compete
* Recall that A could choose any option from 1 to NMatrix in response to B
  + “ could be anything from 1 to NMatrix”
* This can be reflected in chart form but can prove difficult in high Ns:



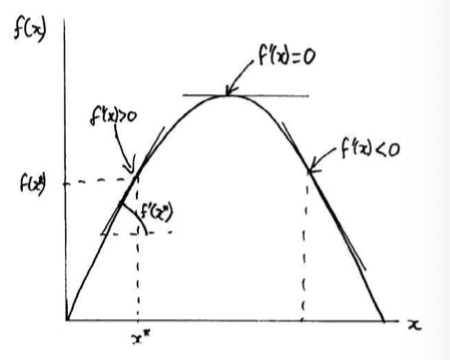
**…**

* + Hence, we see a Payoff Function which is maximised at a point
  + “Find the level of maximiseing firm 1’s payoff for given ”

**4.2: Continuous Strategies**

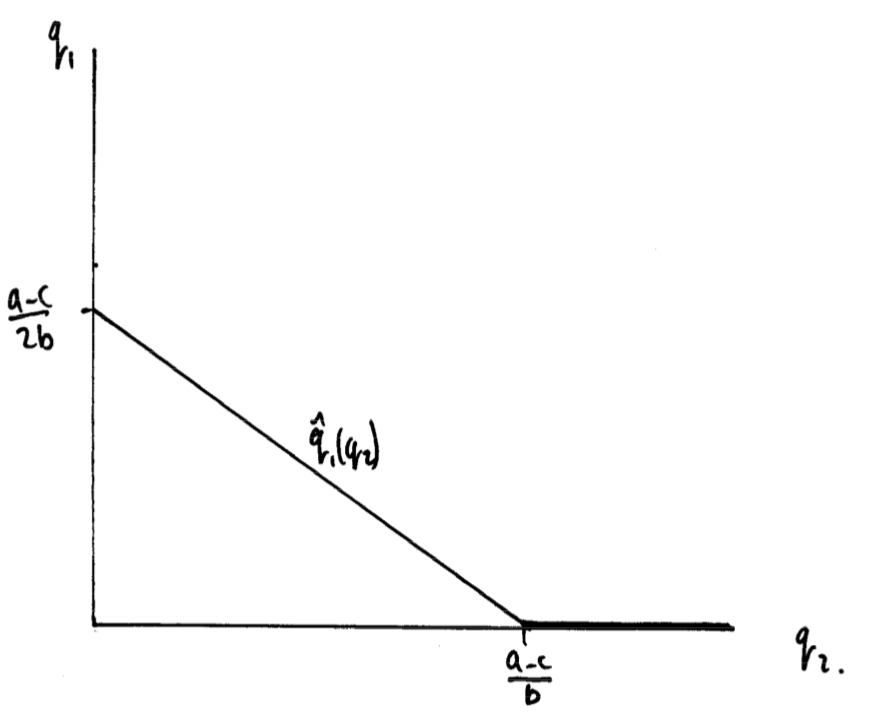
* Too hard to account for all the options (in this case quantities to produce)
* Recall Basic Maths:
  + Function: the level
  + Derivative: the slope of the function
  + Partial Derivative: fix a variable (extract from equation {Hyp. = 0})
* Recall **Rules of Differentiation**:
  + Working towards Payoff Function
  + Just like in the matrixes, fix the opponents option each time to find your best
  + Hence,
  + Fix (the other’s strategy) to observe how varies with
  + Therefore, **partial derivative:**  for fixed
  + Thus, Best Response at {} (peak of function)
  + Note that, if you take the derivative on the **incline** of the function, you can be made better off by doing more. Take the derivative on the **decline**, better off by doing less
  + 

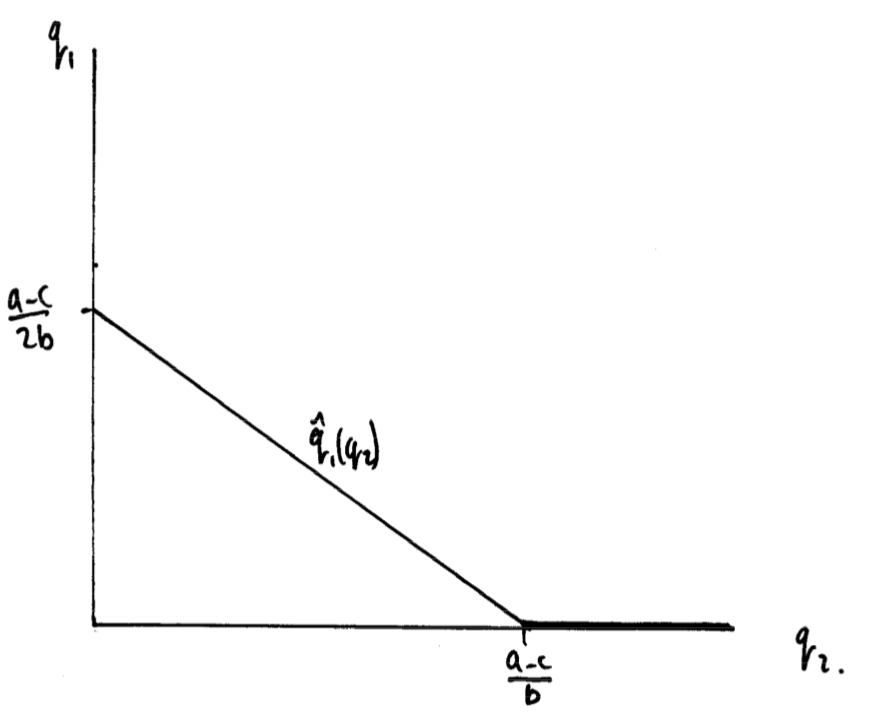


* + Recall: the function :
  + Recall: if
    - **Constant**:
    - **Sum**:
    - **Product:**
    - **Chain:**
    - **Quotient:**
    - **Log:**
  + In Practice:
    - **Power**:
    - **Constant**:
    - **Sum**:
    - **Product:**
    - **Chain:**
    - **Quotient:**

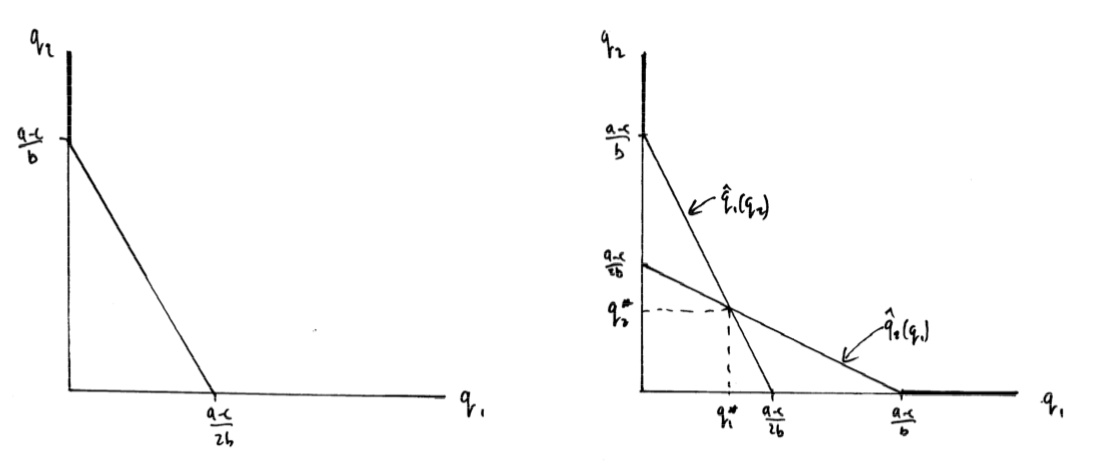
**4.3: Cournot Derivation of Payoff & Reaction (Simultaneous)**

1. Fix **Firm 2**’s action and find my Best Response through Payoff Function
   * Find Payoff Function
   * Partially Derive & {=0} for Best Response with fixed
   * Find best for Reaction Function; Sub for
2. Fix **Firm 1**’s action and find their Best Response through Payoff Function
   * Find Payoff Function
   * Partially Derive & {=0} for Best Response with fixed
   * Find best for Reaction Function; Sub for
3. Find meeting point of **Nash Equilibrium** where both firm’s Reaction Functions meet
4. (Optional) Substitute to find the optimal π for each firm

* **Players**: 2 firms of
* **Strategies**: each firm chooses quantity of
  + For Quantity
* **Payoff**: given supply choices,
  + Marginal Cost of
  + Price
  + Payoff (Profit)
* Working Example for **Firm 1**:
  + For :
  + (Payoff Function of Firm 1)
  + (Fixed )
  + … (Reaction Function of Firm 1)
  + Recall:
  + Note that Reaction Function:
  + Hence, Reaction Function:
  + Output quantity should decline as the opponent’s increases
  + When it reaches 0, leave market
  + Obviously no negative
* Repeat for **Firm 2**:
  + (Payoff Function of Firm 2)
  + (Fixed )
  + or (Reaction Function of Firm 2)
  + Note that Reaction Function:

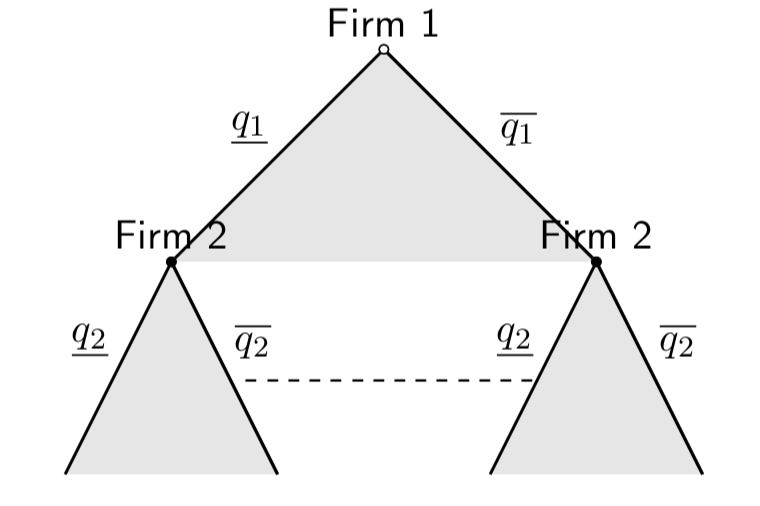


* **Nash Equilibrium**:
  + “The Cournot Equilibrium”
  + Flip Firm 2’s Reaction Function and overlay
  + Seek:
  + “For Firm 1’s *q* which maximises its π given Firm 2’s *q*”
  + “For Firm 2’s *q* which maximises its π given Firm 1’s *q*”



* + Achieved through **Substitution**
  + From:
  + Sub for Firm 2…Sub for π’s…
* Cournot Equilibrium \*\*
  + Hence:
  + So:
  + Thus:
  + ;; ;
* Verify that:
  + Industry Output Between Monopoly and PC:
  + Price is Between Monopoly and PC:
  + Industry Profit Between Monopoly and PC:

**4.4: Stackelberg Leader & Follower (Sequential)**

* The leader implements the first player’s Reaction Function intro their Payoff Function
* First mover advantage as leader gets higher payoff
* Recall from Sequential Games: Backward Induction
* **Linear Demand**:
* **Constant Marginal Costs**:
* **Profits**:
* Firm 1 moves, Firm 2 observes and moves
* Firm 1: Leader; Firm 2: Follower
* Recall Observation:
* Backward Induction:
  + Stage 2: Firm 2 maximises profits given

Firm 2 uses Best Response to whatever Firm 1 produces

* + Stage 1: Firm 1 anticipates reaction of Firm 2 to any decision made

Firm 1 maximises profits given response of Firm 2

Firm 1 chooses point on Firm 2’s Reaction Function which maximises profits

* **Stage 2:**
  + Given what’s the best for Firm 2 (follower) to do? – As Previously…
  + Recall: (Payoff Function)
  + Optimise and {=0}:
  + React: (Reaction Function)
* **Stage 1:** 
  + Firm 1 (Leader) will choose to max. profits taking into account the reaction of the follower
  + Recall: (Payoff)
  + It Knows: if they choose Firm 2 will respond with – account
  + Firm 1 (Leader):

* So Firm 1 Maximises:
* Therefore: (Reaction)
* So Firm 2 (Follower): (Reaction)
* Stackelberg Equilibrium \*\*
  + Idea Is: rather than equilibrium, there is an advantage
  + ; ;
* Stackelberg vs. Cournot:
  + Cournot: ; ; ;
  + Stackelberg: ;

;

; ;

* Hence, First Mover (Firm 1) Advantage!

**5: Applications of Prisoner’s Dilemma**

**5.1: Recalling The Cournot Game**

* An example of a continuous game
* Rather than using Reaction Functions, find Isoprofit Curves
* This is like indifference curves for firms
* Call Firms:
* Strategies:
* Payoff:

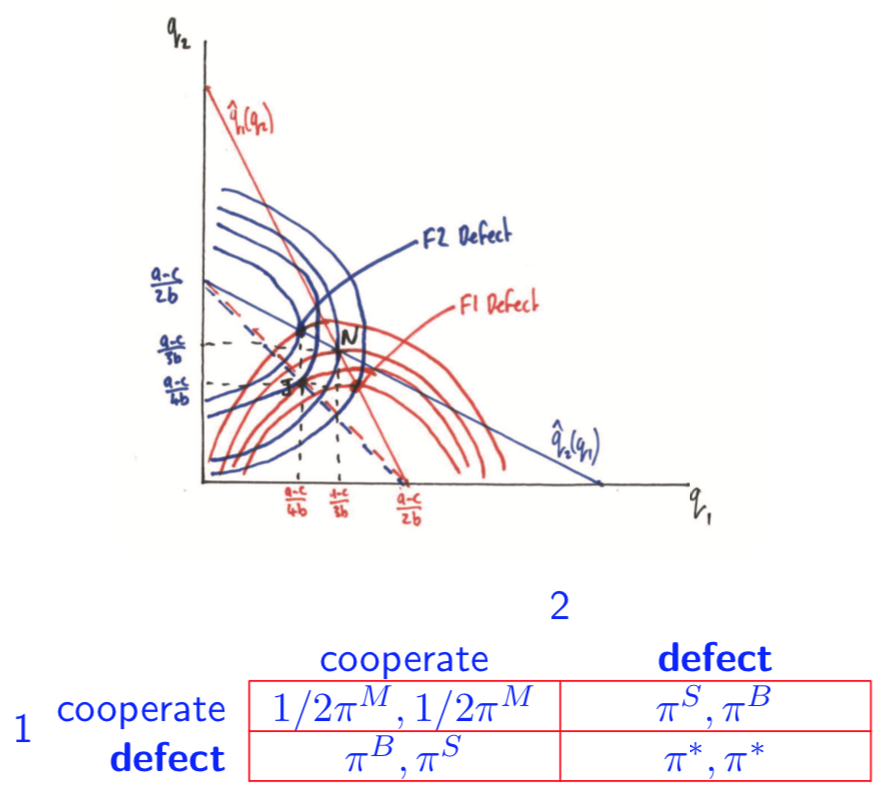
***5.1.1: Typical Reaction Function***

* + From Isoprofit contours
  + Equilibrium at intersection:

***5.1.2: Maximising Joint Profit***

* Hence: assuming ;
  + Makes sense as:

***5.1.3: Will Firms Agree?***

* Will firms agree to produce at half the monopoly output?
* **No**: if firms expect you to produce more than ;
  + Best possible: 🡪 must expand output in excess of Cournot Output
  + Defecting firm: **Bonanza** Payoff
  + Cooperating firm: **Sucker** Payoff
* Hence: Prisoner’s Dilemma

**5.2: Externalities & Strategic Nature**

***5.2.1: Externalities***

* **Negative**:
  + You do more, you lower my payoff (Cournot Game)
  + (Slope of Payoff Function)
* **Positive**:
  + You do more, you lower my payoff
  + (Slope of Payoff Function)

***5.2.2: Strategic Nature***

* **Strategic Substitutes**:
  + Opponent does more of their action: you optimally do less (Reaction Function downward)
  + “with a higher *xj* the optimum is with a lower *xi*”
* **Strategic Compliments**:
  + Opponent does more of their action: you optimally do more (Reaction Function upward)
  + “with a higher *xj* the optimum is with a higher *xi*”

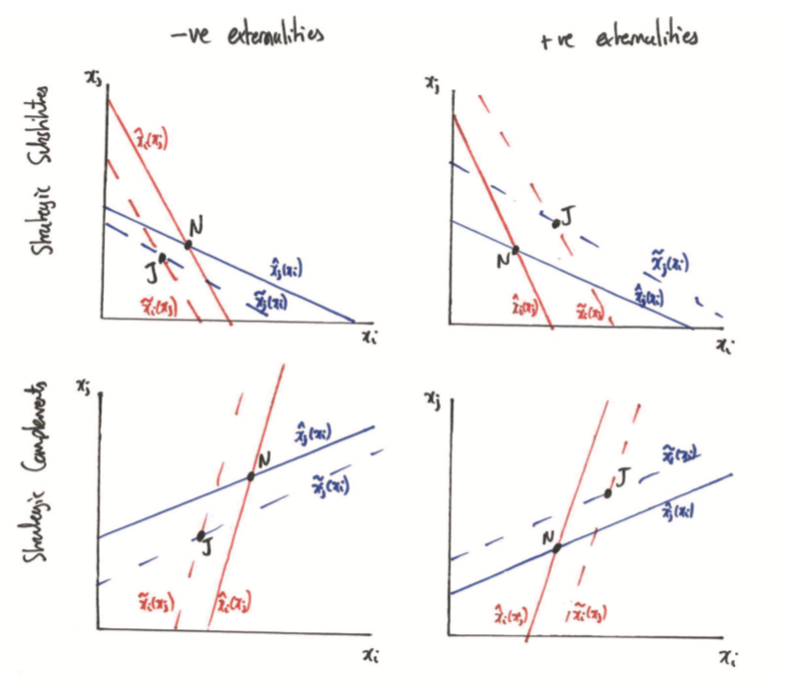
**5.3: Nash Equilibrium in Games**

* When non-cooperative, players optimise self-interest
* Marginal Payoff = 0: &
  + *Note that hat implies function*
* Nash Equilibrium at: Equilibrium
  + Thus Nash Equilibrium actions:

**5.4: Social Planner**

* What happens when they ‘internalise’ the externality?
* Social Planner maximises joint payoff
  + - Chooses to maximise
    - 🡪 these are both Social Optimums

**5.5: Nash Equilibrium vs. Optimum**

* w/ **Positive Externalities**
  + {}
  + So for Social Planner {}
  + So:
* w/ **Negative Externalities**
  + {}
  + So for Social Planner {}
  + So: